Wind Energy Potential Assessment of Great Cumbrae Island Using Weibull Distribution Function

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Authors’ contributions

This work was carried out in collaboration between both authors. Author EYK designed the study, performed the statistical analysis and wrote the first draft of the manuscript. Author JM managed the analysis of the study. Both authors read and approved the final manuscript.

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ABSTRACT

Wind energy is among the fastest growing energy generation technology which is highly preferred alternative to conventional sources of energy. The major Scottish Government target is to deliver 30% of her energy demand by 2020 from renewable sources of energy as well as meeting the emission targets as set under the Scotland Climate Change Act 2009. In this paper, wind energy potential assessment of Great Cumbrae Island was investigated. For this, a ten year mean monthly wind speed at height 50 m obtained from the National Aeronautic Space Administration (NASA) were analysed using the Weibull probability distributions to assess the wind energy potential of Great Cumbrae Island as a clean, sustainable energy resource. Results from the wind-speed model showed that Great Cumbrae Island as high wind-speed site with a mean wind speed of 7.598 m/s and having power density 483.50 W/m². The annual energy captured by four selected horizontal wind turbine models was determined. The result shows that GE 2.0 platform can capture 4.5 GWh energy in a year which is an acceptable quantity for wind energy.

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Keywords: Wind potential; wind power density; Weibull distribution; power generation.

**ABBREVIATIONS**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>NASA</td>
<td>National Aeronautic Space Administration</td>
</tr>
<tr>
<td>GE</td>
<td>General Electricity</td>
</tr>
<tr>
<td>RMSE</td>
<td>Root Mean Square Error</td>
</tr>
<tr>
<td>PNL</td>
<td>Battelle-Pacific Northwest Laboratory</td>
</tr>
</tbody>
</table>

1. **INTRODUCTION**

The world experience shortage of conventional energy resources. The concern about the depletion of fossil fuels reservoirs and global warming drives countries into strong demand for an alternative source of energy especially the renewable sources of energy [1]. There are several renewable energy sources but wind energy is among the fastest growing energy generation technology which is highly preferred alternative to conventional energy sources. This is because wind energy is a clean renewable source of energy without direct gaseous emission into the environment [2,3,4]. The remarkable renewable energy deployment targets in the UK by 2020, the Scottish Government has targeted to produce 100% of her electricity from renewable sources of energy. This corresponds to approximated installed renewable capacity of 14-16 GW [5]. Currently, onshore wind turbines produce 7.3 GW [5]. The major Scottish Government target is to deliver 30% of her energy demand by 2020 from renewable sources of energy as well as meeting the emissions mitigation targets as set under the Scotland Climate Change Act 2009. Producing wind energy has a vital role in meeting this target.

Wind energy utilization depends on average wind speed and wind speed variation. Several techniques to identify the most potential wind sites are available in the literature [6].

In this paper, the standard deviation method has been utilized to determine the value of Weibull parameters [7,8] which are vital in assessing the wind energy potential of a site for power production.

2. **METHODOLOGY**

2.1 **Site Location**

Great Cumbrae Island lies on the coast of Ayrshire and is roughly measures 11.5 square km. The geographical coordinates and the ground height from mean sea level of the Island are shown in Table 1.

<table>
<thead>
<tr>
<th>Location</th>
<th>Height (m)</th>
<th>Longitude (°W)</th>
<th>Latitude (°N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Cumbrae Island</td>
<td>65</td>
<td>4.92</td>
<td>55.75</td>
</tr>
</tbody>
</table>

2.2 **The Weibull Density Function**

The probability density function is given by several authors [8,9,10,11].

\[
f(v) = \frac{k}{c} \left(\frac{v}{c}\right)^{k-1} \exp \left[-\left(\frac{v}{c}\right)^k\right] \text{ for } v > 0 \text{ and } k, c > 0
\] (1)

where \(v\) denotes wind velocity, \(c\) and \(k\) represent the Weibull scale and shape parameter respectively. This function illustrates the probability during which a particular wind speed dominates at a given site with the scale parameter indicating how ‘windy’ the site is [12].

Integrating probability density function gives cumulative density function, \(F(v, k, c)\), which is expressed as [8]:

\[
F(v) = 1 - \exp \left[-\left(\frac{v}{c}\right)^k\right]
\] (2)

Several methods have been proposed to determine Weibull parameters. Moment method (MM) is considered to be the most efficient in estimating Weibull parameters [13]. In this study, the standard deviation method (SDM) was used to determine the accuracy of the moment method. The standard deviation method is expressed as [14,15];
\[ k = \frac{v_m}{\sigma} \cdot 1.086 \quad 1 \leq k \leq 10 \quad (3) \]

\[ c = \frac{v_m k^{2.6}}{0.08 + 0.16 \cdot k^{0.55}} \quad (4) \]

and the moment method expressed as [16];

\[ k = \frac{v_m}{\sigma} \cdot 1.098 \quad (5) \]

\[ c = \frac{v_m}{r(1+k)} \quad (6) \]

where \( \sigma \) stands for the determine wind speed standard deviation. Defined mathematically as:

\[ \sigma = \left( \frac{1}{n-1} \sum_{i=1}^{n} (v_i - v_m)^2 \right)^{0.5} \quad (7) \]

and mean wind speed \( v_m \) expressed as:

\[ v_m = \frac{1}{n} \sum_{i=1}^{n} v_i \quad (8) \]

where \( v_i \) stands for the speed of the wind at time \( i \) and \( n \) denotes the sum of data set. The standard deviation indicates wind speed variation from the mean with low value implying that the wind velocity approaches the mean value. Furthermore, this indicates a high probability of producing a larger amount of power.

2.3 Wind Power Density

The power density of wind flowing through a wind turbine blade of sweep area \( A \) at speed \( v \) increases with the cube of wind speed. Compared to wind speed, the power density of wind is a more reliable parameter in estimating wind power potential of a site. It is expressed as [17]:

\[ P(v) = \frac{1}{2} \rho Av^3 \quad (9) \]

Where \( \rho \) represent air density.

The Weibull density function \( (P_d) \) is expressed as [11]:

\[ P_d = \frac{1}{2} \rho c^3 r \left( 1 + \frac{3}{k} \right) \quad (10) \]

For \( k = 2 \), the Weibull density function (equation 10) becomes Rayleigh density function as follow [14].

\[ P_d = \frac{3}{2} \rho v_m^3 \quad (11) \]

2.4 The Most Frequent Wind Speed

This is the most possible wind speed a given site can experience [14,17]. In a probability distribution function, it is denoted by the peak of the distribution. It is given by:

\[ v_{mp} = c \left( 1 - \frac{1}{k} \right)^{\frac{1}{2}} \quad (12) \]

2.5 Optimal Wind Speed

This represents wind speed that generates the optimum amount of energy using a wind turbine [14,15]. It is mathematically stated as:

\[ v_{op} = c \left( 1 + \frac{2}{k} \right)^{\frac{1}{2}} \quad (13) \]

2.6 Annual Energy Production

The annual energy captured by a wind turbine given by:

\[ E_{Annual} = 0.5 \cdot \rho c_p A v^3 \eta \times 24 \times 365 \quad (14) \]

2.7 Variation of Wind Speed with Height

Wind velocity varies as a power of height governed by the Hellmann exponential laws [18,19,20].

\[ v = v_0 \left( \frac{z}{10m} \right)^{\alpha} \quad (15) \]

Where \( v \) stands for wind speed at tower height \( z \), \( v_0 \) represent wind speed at 10 m height above ground level, while \( \alpha \) representing the Hellman coefficient which is 1/7 for our site [11]. At tower height (80 m) the wind speed was determined as follow:

\[ v = 6.01 \times \left( \frac{80}{10m} \right)^{\frac{1}{7}} = 8.09 m/s \quad (16) \]

3. RESULTS AND DISCUSSION

3.1 Wind Speed Characteristics

Fig. 1 illustrates the Great Cumbrae Island monthly average wind speeds for the period 2008-2010 obtained from Met Office and a 10-year average wind speed from NASA. The analyses were done using OriginPro 8 software. In February 2010, the Island experienced the lowest monthly average wind speed of 4.0 ms\(^{-1}\) and the highest 7.9 ms\(^{-1}\) was experienced in March 2008. The Island received annual
average wind speeds of 6.08, 5.79 and 5.06 m s\(^{-1}\) for 2008, 2009 and 2010 respectively with a 10-year average wind speed varying from 4.8 m s\(^{-1}\) in July to 7.3 m s\(^{-1}\) in January. Based on the rule of thumb for annual average wind speeds \[16], Great Cumbrae Island is suitable for grid connected wind farm for power production.

3.2 Great Cumbrae Island Weibull Parameters

Fig. 2 shows the Weibull probability function spreading over a wider range. This distribution shows that Great Cumbrae Island mostly experiences wind speed higher than 5.0 m s\(^{-1}\).

**Fig. 1. Monthly average wind speed at 10 m elevation for Great Cumbrae Island**

**Fig. 2. The Weibull probability function**
Fig. 3 shows a high probability (0.55) of the Island to experience annual average wind speed of more than 5.0 ms\(^{-1}\).

3.3 Weibull Parameters and Power Density Analysis

Table 2 shows the monthly wind speed characteristics of Great Cumbrae Island. The Weibull parameters were determined using equations (3-6). Based on Weibull distribution, equation (10) was used to determine the wind power density.

The parameter k and c, ranged from 2.102-2.495 and 10.383-6.556 m/s, respectively in different months of the year. The high values of c imply high values of \(v_m\) and the wide range distribution of the probability function. The Island experienced wind with the highest power density of 834.252 \(W/m^2\) in January and the lowest power density of 211.797 \(W/m^2\) in July.

According to PNL wind power classification [21] shown in Table 4, the monthly average wind power density of Great Cumbera Island mostly falls into class 4.

To test the accuracy of moment method with regard to the standard deviation method, the root means square error (RMSE) was used. This error is mathematically expressed as [22,23]:

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{N} (y_i - x_i)^2}{N}}
\]

Where \(y_i\) denotes the Weibull parameter value established using moment method, \(x_i\) represent the Weibull parameter value determined by the standard deviation method and \(N\) is the sum of data points.

The result of the statistical error analysis illustrated in Table 3 shows that the Moment Method is an accurate method because it has a small RMSE.

3.4 Wind Turbine Selection and Annual Energy Production

In this section, the production of four horizontal wind turbine models were calculated. Table 5 illustrates the characteristic of each model. The models were selected based on their tower height.

The annual power output captured by each wind turbine model was calculated using equation (11-13) With an efficiency of 50% (\(\eta\)) and capacity factor 30% (\(c_p\)) [25].

Fig. 4 shows the annual energy captured by each turbine model at a tower height of 80 m. GE 2.0 platform captured the highest energy and for this reason it is an ideal turbine model for the Island. About 4.5 GWh energy can be obtained in a year which is an acceptable quantity for wind energy. With the installation of a wind farm at the Island, more energy can be generated.

### Table 2. Wind speed characteristics for Great Cumbrae Island

<table>
<thead>
<tr>
<th>Months</th>
<th>(v_m)</th>
<th>(\sigma)</th>
<th>SDM</th>
<th>MM</th>
<th>(v_{op})</th>
<th>(v_{mp})</th>
<th>(P_d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apr</td>
<td>7.106</td>
<td>3.052</td>
<td>8.012</td>
<td>2.504</td>
<td>8.009</td>
<td>2.495</td>
<td>10.128</td>
</tr>
<tr>
<td>Jul</td>
<td>5.990</td>
<td>2.638</td>
<td>6.757</td>
<td>2.437</td>
<td>6.556</td>
<td>2.427</td>
<td>8.640</td>
</tr>
</tbody>
</table>

### Table 3. Statistical error analysis summary

<table>
<thead>
<tr>
<th>Weibull parameter</th>
<th>Statistical methods</th>
<th>Statistical test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MM</td>
<td>SDM</td>
</tr>
<tr>
<td>(k)</td>
<td>2.314</td>
<td>2.324</td>
</tr>
<tr>
<td>(c (ms^{-1}))</td>
<td>8.558</td>
<td>8.576</td>
</tr>
</tbody>
</table>
Fig. 3. The Weibull frequency distribution

Fig. 4. Annual energy produced from wind turbines at tower height 80 m

Table 4. PNL wind power classification for 50 m elevation [24]

<table>
<thead>
<tr>
<th>Wind power class</th>
<th>Potential</th>
<th>Wind speed (m/s)</th>
<th>Wind Power (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Poor</td>
<td>0.0 &lt; v ≤ 5.6</td>
<td>0 &lt; P_d ≤ 200</td>
</tr>
<tr>
<td>2</td>
<td>Marginal</td>
<td>5.6 &lt; v ≤ 6.4</td>
<td>200 &lt; P_d ≤ 300</td>
</tr>
<tr>
<td>3</td>
<td>Moderate</td>
<td>6.4 &lt; v ≤ 7.0</td>
<td>300 &lt; P_d ≤ 400</td>
</tr>
<tr>
<td>4</td>
<td>Good</td>
<td>7.0 &lt; v ≤ 7.5</td>
<td>400 &lt; P_d ≤ 500</td>
</tr>
<tr>
<td>5</td>
<td>Excellent</td>
<td>7.5 &lt; v ≤ 8.0</td>
<td>500 &lt; P_d ≤ 600</td>
</tr>
<tr>
<td>6</td>
<td>Excellent</td>
<td>8.0 &lt; v ≤ 8.8</td>
<td>600 &lt; P_d ≤ 800</td>
</tr>
<tr>
<td>7</td>
<td>Excellent</td>
<td>More than 8.8</td>
<td>More than 800</td>
</tr>
</tbody>
</table>
Table 5. Characteristic wind turbine [26,27,28,29]

<table>
<thead>
<tr>
<th>Model</th>
<th>Rated Power (kW)</th>
<th>Tower height (m)</th>
<th>Rotor diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>V110-Vestas</td>
<td>2,000</td>
<td>80</td>
<td>110</td>
</tr>
<tr>
<td>V90-Vestas</td>
<td>2,000</td>
<td>80</td>
<td>90</td>
</tr>
<tr>
<td>GE 2.0 platform</td>
<td>2,000</td>
<td>80</td>
<td>116</td>
</tr>
<tr>
<td>Siemens</td>
<td>2,300</td>
<td>80</td>
<td>101</td>
</tr>
</tbody>
</table>

4. CONCLUSION

The following is the summary of the most crucial findings of this study:

1. The annual Weibull shape parameter values at 50 m elevation ranged between 2.102 and 2.495 with an average value of 2.314 see Table 2. This value indicates a site with a slight hourly mean wind speed variation about the yearly wind speed resulting in a good quality wind power production.

2. Great Cumbrae Island mostly experiences annual average wind speed of more than 5.0 \( m/s \). This makes the Island suitable for grid connected wind farm for power production.

3. From the assessment conducted it was found that Great Cumbrae Island receives wind of power density of 483.50 W/m\(^2\). According to PNL wind power classification, Great Cumbrae Island falls into class 4. It is therefore a suitable site for installation of the wind farm for power production.

4. The calculated annual energy captured by four wind turbines model shows a high probability of generating higher than 4.5 GWh energy with the installation of a wind farm at the Island.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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